

Generalized rotating-wave approximation to biased qubit-oscillator systems

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We present the generalized rotating-wave approximation to biased qubit-oscillator systems including the counter-rotating interactions analytically. Concise analytical expressions for all eigen-solutions are explicitly given. Our analytical results for the spectrum of the flux qubit coupled to superconducting oscillators can account excellently for the experimental observations. The resulting energy levels for the different biased case agree well with the numerical exact ones in a wide range of coupling strength. The dynamics of the qubit are also calculated, the validity of the presented analytical approximation is shown to be very well. The present theoretical approach can be easily implemented, and successfully applied to the present and near future superconducting qubit-oscillator experiments with a biased qubit for accessible coupling strengths.

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I. INTRODUCTION

The two-level system (or qubit) and the harmonic oscillator are attractive components of quantum systems due to its interesting application in two-level atoms coupled to optical or microwave cavities [1, 2], superconducting qubits coupled to superconducting resonators [3–7]. In the early work on cavity quantum electrodynamics (QED), the qubit-oscillator coupling strengths g achieved were much smaller than the cavity transition frequency ω , $g/\omega \sim 0.001$. All experiments can be well described by the Jaynes-Cummings model with rotating-wave approximations (RWA) [8].

In recent circuit QED, where superconducting artificial two-level atoms are coupled to on-chip cavities, the exploration of quantum physics has greatly evolved in the ultrastrong coupling regime, where the atom-cavity coupling strength is comparable to the cavity transition frequency, $g/\omega \sim 0.1$ [4, 5, 9, 18, 20]. It is evident for the breakdown of the RWA and the counter-rotating terms are expected to take effect. There have been numerous of theoretical studies on the qubit and oscillator system finding new phenomena in the ultrastrong coupling regime [10–12] and deep strong coupling regime $g/\omega > 1$ [13, 14]. Recent, Irish [12] has put forth a generalized rotating-wave approximations (GRWA) associated with the adiabatic approximation and the standard RWA, which works well in a wide range coupling regime. However, it has not been applied to study biased qubit-oscillator systems with a static biased qubit. In circuit QED systems, the magnetic energy bias manipulation of the flux qubit is associated with the circulating current in the qubit loop and the applied magnetic flux [9, 18, 20]. Taking the qubit's bias into account, Grifoni et al. [11] present the Van Vleck perturbation (VVP) theory beyond the RWA to treat analytically the two-level system coupled to a harmonic oscillator. However, it gives unphysical cross-ings in the energy levels in weak coupling region for pos-

itive detuning.

In this paper we develop the generalized rotating-wave approximations to biased qubit-oscillator systems in the ultrastrong coupling regime, so-called the biased generalized rotating-wave approximations (BGRWA). It is improved to evaluate a renormalized Hamiltonian with the mathematical structure of the ordinary RWA in the finite bias case, giving the analytical eigenvalues and eigenfunctions. It is on the equal foot of the GRWA and can recover the results of GRWA for an unbiased qubit. Following the circuit QED experiment observation in the ultrastrong coupling regime, we provide an concise spectrum analytically using fitting parameters in the experiment. And we discuss the validity of the energy levels of the system by comparing with those obtained by the VVP method and numerical exact diagonalization. Furthermore, we also study the qubit dynamics in the ultrastrong coupling regime to confirm the effectiveness of the BGRWA.

The paper is outlined as follows. In Sec. II, the analytical solution by the BGRWA to the biased qubit-oscillator system are evaluated in detail, giving the eigenenergies and eigenstates. And the spectrum is shown with our analytical expression using the experiment parameters. In Sec. III, we calculate the dynamics of the qubit in the finite bias case. Finally, a brief summary is given in Sec. IV.

II. ANALYTICAL SOLUTION

The Hamiltonian of the qubit can be written as $H_q = -(\varepsilon\sigma_x + \Delta\sigma_z)/2$, where σ_x and σ_z are Pauli matrixes; Δ is the tunneling parameter between the upper level $|+z\rangle$ and the lower level $|-z\rangle$ in the basis of σ_z ; ε is the magnetic energy bias related to the circulating current in the qubit loop and the applied magnetic flux [18, 20]. In weak coupling regime, where the interaction strength g

exceeds the cavity and qubit loss rates, the RWA can be applied and the system can be described by the Jaynes-Cummings type Hamiltonian for zero bias as ($\hbar = 1$)

$$H = \frac{\Delta}{2}\sigma_z + \omega a^\dagger a + g(a^\dagger \sigma_- + a \sigma_+), \quad (1)$$

where a^\dagger and a are the creation and annihilation operators for the oscillator. It can be solved analytically in a closed form in the basis $|n\rangle|+z\rangle$ and $|n+1\rangle|-z\rangle$, where the qubit states $|\pm z\rangle$ are eigenstates of σ_z and the oscillator states $|n\rangle$ ($n = 0, 1, 2, \dots$) are Fock states. The ground state obtained is $|\psi_g\rangle = |0\rangle|-z\rangle$ for weak coupling. However, experimental advances are pushing into the ultrastrong coupling region, where g approaches to the qubit or oscillator frequencies, the RWA no longer holds [9, 18, 20]. Thus, the qubit-oscillator counter-rotating interaction $a^\dagger \sigma_+ + a \sigma_-$ needs to be taken into account.

Under a rotation around the y axis with the angle $\pi/2$, the Hamiltonian of the qubit-oscillator system including the counter-rotating terms reads

$$H = -\frac{\Delta}{2}\sigma_x - \frac{\varepsilon}{2}\sigma_z + \omega a^\dagger a + g(a^\dagger + a)\sigma_z. \quad (2)$$

Using the unitary transformation $U = \exp[-\frac{g}{\omega}\sigma_z(a - a^\dagger)]$, we obtain the transformed Hamiltonian $H' = U^\dagger H U = H_0 + H_1$, consisting of

$$\begin{aligned} H_0 &= \omega a^\dagger a - g^2/\omega - \frac{\varepsilon}{2}\sigma_z, \\ H_1 &= -\frac{\Delta}{2}\{\sigma_x \cosh[\frac{2g}{\omega}(a^\dagger - a)] + i\sigma_y \sinh[\frac{2g}{\omega}(a^\dagger - a)]\}. \end{aligned} \quad (3)$$

Recently, many groups have been devoted to solve qubit-oscillator systems by different transformations [11, 12, 15–17]. We now extend the generalized approximation by Irish [12] to the biased qubit-oscillator system. The simplicity of the approximation is based on its close connection to the standard RWA. Consequence, the terms retained in H_1 correspond to energy-conserving one-excitation terms, just as the standard RWA. When $\cosh[\frac{2g}{\omega}(a^\dagger - a)]$ is expanded as $1 + \frac{1}{2!}[\frac{2g}{\omega}(a^\dagger - a)]^2 + \frac{1}{4!}[\frac{2g}{\omega}(a^\dagger - a)]^4 + \dots$, it is performed by keeping the terms containing the number operator $a^\dagger a = n$ with the coefficient $G_0(n)$

$$\begin{aligned} G_0(n) &= \langle n | \cosh[\frac{2g}{\omega}(a^\dagger - a)] | n \rangle \\ &= \exp(-2g^2/\omega^2) L_n(4g^2/\omega^2), \end{aligned} \quad (5)$$

where L_n are the Laguerre polynomials. The other excitation terms as well as $a^{\dagger 2}$, a^2, \dots , which are accounted for multi-photon process, are neglected within this approximation. Similarly, by expanding $\sinh[\frac{2g}{\omega}(a^\dagger - a)] = \frac{2g}{\omega}(a^\dagger - a) + \frac{1}{3!}[\frac{2g}{\omega}(a^\dagger - a)]^3 + \frac{1}{5!}[\frac{2g}{\omega}(a^\dagger - a)]^5 + \dots$, the one-excitation terms are kept as $F_1(n)a^\dagger - aF_1(n)$ with

the coefficient $F_1(n)$ to be determined. Since the terms $aF_1(n)$ and $F_1(n)a^\dagger$ involve creating and eliminating a single photon of the oscillator, it can be evaluated as

$$\begin{aligned} F_1(n) &= \frac{1}{\sqrt{n+1}} \langle n+1 | \sinh\left[\frac{2g}{\omega}(a^\dagger - a)\right] | n \rangle \\ &= \frac{g\Delta}{n+1} e^{-2g^2/\omega^2} L_n^1(4g^2/\omega^2). \end{aligned} \quad (6)$$

Since the higher-order terms of H_1 are discarded, we construct an effective Hamiltonian $H' = H'_0 + H'_1$,

$$\begin{aligned} H'_0 &= \omega a^\dagger a - g^2/\omega - \frac{\Delta\eta}{2}\sigma_x - \frac{\varepsilon}{2}\sigma_z, \\ H'_1 &= -\frac{\Delta}{2}[G_0(n) - \eta]\sigma_x - i\frac{\Delta}{2}\sigma_y F_1(n)(a^\dagger - a), \end{aligned} \quad (7)$$

where the parameter η is defined as $\eta = G_0(0)$.

Since the qubit and oscillator are decoupled in H'_0 and its qubit part can be diagonalized by a second unitary transformation $S = \begin{pmatrix} u & v \\ v & -u \end{pmatrix}$ with $u = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{\varepsilon}{y}}$, $v = \frac{1}{\sqrt{2}}\sqrt{1 + \frac{\varepsilon}{y}}$, and $y = \sqrt{\varepsilon^2 + \Delta^2\eta^2}$. The diagonalized H'_0 is

$$\tilde{H}_0 = S^\dagger H'_0 S = \omega a^\dagger a - g^2/\omega + \frac{1}{2}\sqrt{\varepsilon^2 + \Delta^2\eta^2}\sigma_z, \quad (8)$$

where the tunneling parameter is renormalized by $\frac{1}{2}\sqrt{\varepsilon^2 + \Delta^2\eta^2}$. And the H'_1 is transformed into

$$\begin{aligned} \tilde{H}_1 &= S^\dagger H'_1 S \\ &= \frac{\Delta^2\eta[G_0(n) - \eta]}{2\sqrt{\varepsilon^2 + \Delta^2\eta^2}}\sigma_z - \frac{\Delta}{2}F_1(n)(\sigma_+ - \sigma_-)(a^\dagger - a) \\ &\quad - \frac{\Delta\varepsilon[G_0(n) - \eta]}{2\sqrt{\varepsilon^2 + \Delta^2\eta^2}}\sigma_x. \end{aligned} \quad (9)$$

To establish a mathematical structure of the standard RWA, the Hamiltonian with BGRWA can be evaluated approximately in the form

$$\begin{aligned} H_{BGRWA} &= \omega a^\dagger a - g^2/\omega + \frac{\varepsilon^2 + \Delta^2\eta[G_0(n) - \eta]}{2\sqrt{\varepsilon^2 + \Delta^2\eta^2}}\sigma_z \\ &\quad + R_r(a^\dagger \sigma_- + a \sigma_+), \end{aligned} \quad (10)$$

where the effective coupling strength is renormalized into $R_r = \frac{\Delta}{2}F(n)$, depending on the parameters Δ and g . Combining the transformation, the new Hamiltonian with the mathematical structure of the ordinary RWA contains the counter-rotating interactions, which play a important role in the ultrastrong coupling regime.

In the zero bias case, the GRWA by Irish [12] works for arbitrarily large coupling and for a wide range of detuning values. We extend the derivation to the BGRWA, which can be applied to zero- and finite-bias qubit-oscillator systems. As it can be seen from (10), the

effective Hamiltonian including the counter-rotating interactions is a generalization of the energy-conserving terms $R_r(a^\dagger\sigma_- + a\sigma_+)$ in the usual RWA Hamiltonian(1). Our approximation exhibits the simplicity as well as the

GRWA and is expected to extend the range of validity to ultrastrong coupling strengths for a finite-biased qubit.

One can easily diagonalize the Hamiltonian in the basis of $|+z, n\rangle$ and $|-z, n+1\rangle$

$$H_{BGRWA} = \begin{pmatrix} \omega n - g^2/\omega + \frac{\varepsilon^2 + \Delta^2 \eta G_0(n)}{2\sqrt{\varepsilon^2 + \Delta^2 \eta^2}} & R_r(n)\sqrt{n+1} \\ R_r(n)\sqrt{n+1} & \omega(n+1) - g^2/\omega - \frac{\varepsilon^2 + \Delta^2 \eta G_0(n+1)}{2\sqrt{\varepsilon^2 + \Delta^2 \eta^2}} \end{pmatrix}. \quad (11)$$

It is straightforward to obtain the eigenvalues

$$E_n^{\pm, BGRWA} = \omega(n + \frac{1}{2}) - g^2/\omega + \frac{\Delta^2 \eta}{4\sqrt{\varepsilon^2 + \Delta^2 \eta^2}} e^{-2g^2/\omega^2} [L_n(4g^2/\omega^2) - L_{n+1}(4g^2/\omega^2)] \\ \pm \{ [\frac{\omega}{2} - \frac{2\varepsilon^2 + \Delta^2 \eta e^{-2g^2/\omega^2} [L_n(4g^2/\omega^2) + L_{n+1}(4g^2/\omega^2)]}{4\sqrt{\varepsilon^2 + \Delta^2 \eta^2}}]^2 + \frac{g^2 \Delta^2 e^{-4g^2/\omega^2}}{\omega^2(n+1)} [L_n^1(4g^2/\omega^2)]^2 \}^{1/2}, \quad (12)$$

and the corresponding eigenfunctions

$$|\varphi_n^+\rangle = \cos \frac{\theta}{2} |n\rangle | +z \rangle + \sin \frac{\theta}{2} |n+1\rangle | -z \rangle, \quad (13)$$

$$|\varphi_n^-\rangle = \sin \frac{\theta}{2} |n\rangle | +z \rangle - \cos \frac{\theta}{2} |n+1\rangle | -z \rangle, \quad (14)$$

where $\cos \theta = \frac{\delta}{\sqrt{\delta^2 + 4R_r^2(n+1)}}$ and $\delta = \frac{2\varepsilon^2 + \Delta^2 \eta [L_n(4g^2/\omega^2) + L_{n+1}(4g^2/\omega^2)]}{2\sqrt{\varepsilon^2 + \Delta^2 \eta^2}} - \omega$. In the case of $\varepsilon = 0$, the eigenvalues in Eq.(12) are reduced to GRWA form [12].

The ground-state eigenenergy is

$$E_g^{BGRWA} = -\frac{1}{2} \sqrt{\varepsilon^2 + \Delta^2 \eta^2} - g^2/\omega, \quad (15)$$

with the ground state $|-z, 0\rangle$.

Attributing to the advanced experiment in qubit-oscillator systems, there are observations of the spectrum in the ultrastrong coupling regime [18], which was fitting numerically by exact diagonalization in the Fock basis [18] and in the coherent-state basis [19]. In the experiment of a flux qubit coupled to an superconducting oscillator, the bias parameter $\varepsilon = 2I_p(\Phi - \Phi_0/2)$ with I_p the persistent current in the qubit loop, Φ an externally applied magnetic flux, and $\Phi_0 = \hbar/2e$ the flux quantum. Fig. 1 shows the spectrum of the system using Eqs.(12) and (15), with fitted parameters of the experimental results $g/2\pi = 0.82$, $\omega/2\pi = 8.13$, $\Delta = 4.25$ and $I_p = 510nA$. No significant difference can be observed

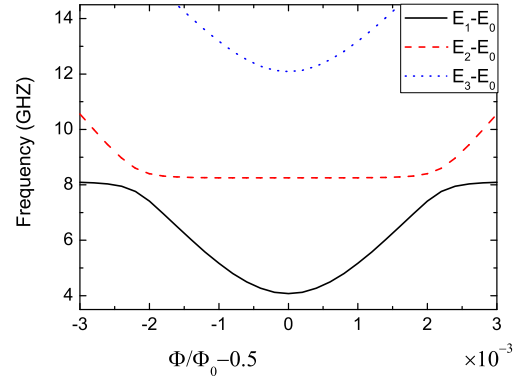


FIG. 1: Spectrum of the qubit coupled to an oscillator obtained from Eqs.(12) and (15) using the experiment parameters in Ref [18].

between our results and experiment ones [18]. It demonstrates that our BGRWA will have a remarkable useful application in the future experiment since large strong coupling strength will be accessed.

To the best of our knowledge, the analytical expressions Eqs.(12)-(15) for the present BGRWA is the most simple one among the existing analytical works. It should become potentially effective tools in the study of the superconducting qubit-oscillators where the biased parameter is mainly manipulated, if the validity is verified in

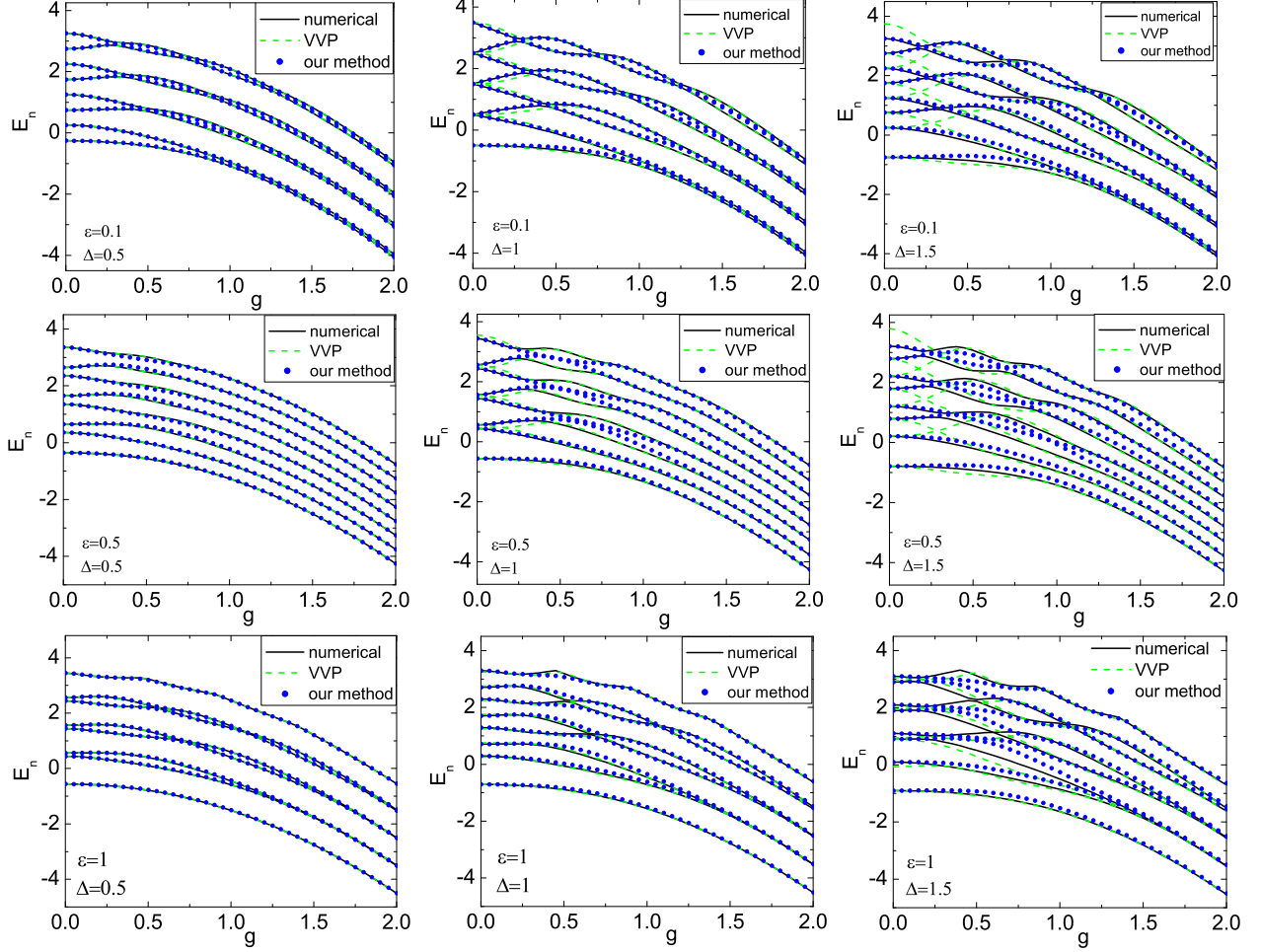


FIG. 2: Energy levels E_n as a function of coupling strength g for different bias $\varepsilon = 0.1, 0.5$ and 1 (from top to bottom) with the tunneling parameter $\Delta = 0.5, 1$ and 1.5 . We compare the eigenvalues in Eq. (12) (solid dot) with those obtained by the numerical exact diagonalization method (solid line) and VVP eigenvalues in Eq. (16) (dashed line). We set $\omega = 1$.

general. To show the validity of the BGRWA in the biased qubit-oscillator system, we present a detailed dis-

cussion on the energy spectrum and consider eigenvalues obtained by the VVP method, which is written as [11]

$$\begin{aligned}
 E_m^\pm = & (m + \frac{l}{2})\omega - \frac{g^2}{\omega} + \frac{1}{2} \sum_{k=0, k \neq m \pm l} \left(\frac{D_{mk}^2}{\varepsilon + (m-k)\omega} - \frac{D_{nk}^2}{\varepsilon + (k-n)\omega} \right) \\
 & \pm \frac{1}{2} \sqrt{[\varepsilon - l\omega + \Delta^2 \sum_{k=0, k \neq m \pm l} (\frac{D_{mk}^2}{\varepsilon + (m-k)\omega} + \frac{D_{nk}^2}{\varepsilon + (k-n)\omega})]^2 + 4D_{mn}^2}, \\
 & (n = m + l, l \geq 0)
 \end{aligned} \tag{16}$$

where $D_{mn} = \frac{\Delta}{2}(-1)^m(2g)^{n-m}e^{-2g^2}\sqrt{\frac{m!}{n!}}L_m^{n-m}(4g^2/\omega^2)$.

The m th eigenvalues E_m^\pm is a mixture of the oscillator

levels m and l . Note that the ambiguity of the value of l , which should be selected by experience to give better results. It works well for large bias ε and strong coupling. By comparing with our analytical eigenenergies in Eq. (12), our approach can be more easily implemented. The detailed comparison with our approach is present in the following for different coupling strength g and detuning parameter Δ/ω .

Fig. 2 shows the first eight energy levels against the coupling strength g for three different values of bias ε and tunneling parameter Δ for energy scale $\omega = 1$. For negative detuning $\Delta = 0.5$, our analytical method and the VVP show good agreement with the numerical results from weak coupling regime to strong coupling regime for different bias values $\varepsilon = 0.1, 0.5$ and 1 , as shown in Fig. 2(left panel). At resonance $\Delta = 1$ (middle panel), our analytical solutions agree well with the numerical results at $g < 0.5$, which is just the present accessible coupling regime $g < 0.12$ [9] and will be first possibly accessible regime in the future experiments. Just at this practically interesting coupling regime, the VVP results deviates considerably. The deviations from the numerical results in the intermediate coupling regime is due to the dominated influence of the higher order terms neglecting in the transformed hamiltonian in Eq. (7) where more photon excitations play roles. In the case of positive detuning $\Delta = 1.5$, there is an improvement by our analytical results in weak coupling regimes with its comparison to the VVP ones, as shown in Fig. 2 (right panel). Especially, for $\varepsilon = 0.1$ and 0.5 , the results of the VVP in the weak coupling regime is qualitatively incorrect with a unphysical crossing, but that of our BGRWA solutions is still in agreement with the numerically exact one. Therefore we can say that the BGRWA approach, which takes into account the effect of counter-rotating terms, is quite simple and can give accurate analytical solution in the biased case.

III. DYNAMICS OF THE QUBIT

In the original Hamiltonian (2), the excited wavefunctions without RWA can be evaluated by unitary transformation $|\Psi_n^\pm\rangle = U^\dagger S^\dagger |\varphi_n^\pm\rangle$ in the following

$$\begin{aligned} |\Psi_n^+\rangle^{BGRWA} &= (u \cos \frac{\theta}{2} |n\rangle_{\frac{g}{\omega}} + v \sin \frac{\theta}{2} |n+1\rangle_{\frac{g}{\omega}}) | + x \rangle \\ &+ (v \cos \frac{\theta}{2} |n\rangle_{\frac{-g}{\omega}} - u \sin \frac{\theta}{2} |n+1\rangle_{\frac{-g}{\omega}}) | - x \rangle, \\ |\Psi_n^-\rangle^{BGRWA} &= (u \sin \frac{\theta}{2} |n\rangle_{\frac{g}{\omega}} - v \cos \frac{\theta}{2} |n+1\rangle_{\frac{g}{\omega}}) | + x \rangle \\ &+ (v \sin \frac{\theta}{2} |n\rangle_{\frac{-g}{\omega}} - u \cos \frac{\theta}{2} |n+1\rangle_{\frac{-g}{\omega}}) | - x \rangle, \end{aligned} \quad (17)$$

where the qubit states $|\pm x\rangle$ are eigenstates of σ_x and the oscillator states $|n\rangle_{\pm g/\omega} = e^{\mp g/\omega(a-a^\dagger)}|n\rangle$ are displacement-shift Fock states, so-called coherent states.

And the ground-state wavefunction is obtained by

$$|\Psi_g\rangle^{BGRWA} = v|e\rangle e^{-g/\omega(a-a^\dagger)}|0\rangle - u|g\rangle e^{g/\omega(a-a^\dagger)}|0\rangle. \quad (18)$$

We now contrast the dynamical evolution of $\sigma_z(t)$ to present the validity of the analytical solutions. The initial state is assumed as $|\varphi(0)\rangle = |\uparrow\rangle|0\rangle$. With the eigenbasis $\{|\Psi_n\rangle^{BGRWA}\}$ and eigenenergies $\{E_n^{BGRWA}\}$, the dynamical wavefunction of the Hamiltonian without RWA (2) can be expressed as

$$\begin{aligned} |\varphi(t)\rangle &= e^{-iHt}|\varphi(0)\rangle \\ &= \sum_n e^{-itE_n^{BGRWA}} |\Psi_n\rangle^{BGRWA} \langle\Psi_n|\varphi(0)\rangle. \end{aligned} \quad (19)$$

We calculate the time evolution of $\langle\sigma_z(t)\rangle = \langle\varphi(t)|\sigma_z|\varphi(t)\rangle$ using our method presented in this paper without RWA for the biased case. The expectation value of $\langle\sigma_z(t)\rangle$ is plotted in Fig. 3 for $\varepsilon = 0.1$ (upper panel) and $\sqrt{0.5}$ (lower panel) for different coupling strength $g = 0.1, 0.2, 0.5$. For comparison, the results of exact diagonalization and those of VVP are also shown. One finds that the analytical dynamical evolution agrees well with the numerical exact ones, much better than those obtained by VVP. It follows that the contribution of the counter-rotating interaction is reasonably considered in the BGRWA analytical solution. Thus, the BGRWA can be applied validly in the dynamical evolution in a wide range of coupling strength.

IV. CONCLUSION

We have presented the analytical solution to the qubit-oscillator system without the RWA for the finite-biased qubit by the BGRWA. The simplified approach takes into account the counter-rotating interaction well but still keeps the simple mathematical structure of the ordinary RWA. It facilitates us to give the analytical eigenfunctions and eigenenergies simply, which can also recover the results of GRWA for zero bias. Our approximation has extended the validity to the ultrastrong coupling regime. The spectrum obtained by the analytical expression are in good consistent with experiment observations. And we have compared with energy levels with the VVP and numerical results, exhibiting a wide range of validity to large coupling strengths $g < 0.5$. One finds that there is an improvement of the energy levels of ground and lower-lying excited states by compared with VVP results. Especially, our analytical energy levels fit well with numerical results in the positive detuning region, where the VVP is invalid. Moreover, our BGRWA dynamical evolutions quantitatively agree well with the numerical exact ones for weak to ultra-strong coupling. The present analytical approach is simple to be implemented and should be very useful for the present superconducting qubit-oscillator coupling systems with $g < 0.12$. More over,

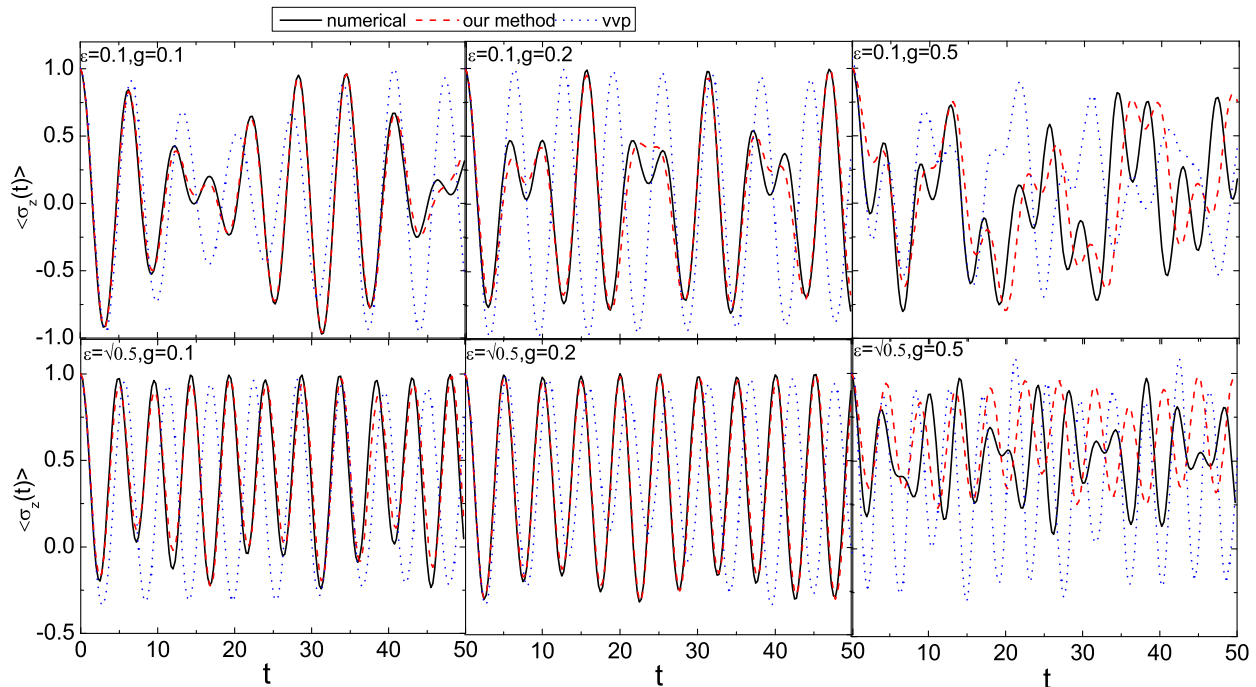


FIG. 3: Time evolution of $\sigma_z(t)$ for $\varepsilon = 0.1$ (upper panel) and $\sqrt{0.5}$ (low panel) for different coupling strength $g = 0.1, 0.2, 0.5$ on resonance. The dashed line is our BGRWA calculation. The solid and the dotted line are the numerically exact result and the VVP one, respectively. The evolution starts with a vacuum state $|0\rangle$ and an excited spin $|e\rangle$.

there is still wide room $g < 0.5$ in BGRWA to accurately account for the future experiments in the same field where the coupling strength is further increased to explore novel phenomena beyond the RWA. Finally, it is suggested that the present approach can be extended to more complicated problems, such as biased spin-boson systems.

Acknowledgments

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